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OPERATIONS RESEARCH IN TRANSIT: A SURVEY

by

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ABSTRACT

The purpose of this paper is to briefly review how operations research techniques have been used to solve planning and operational problems in transit authorities. This review is not exhaustive, but emphasizes operations research techniques that have been applied or that, in our opinion, have potential applications.

Key words : operations research, transit, scheduling.

RÉSUMÉ

Nous présentons brièvement les principales applications de la recherche opérationnelle aux problèmes de transport collectif. Nous touchons aussi bien les problèmes de planification que d'exploitation. Nous ne prétendons pas que cette revue est complète. De plus, nous nous sommes volontairement restreints aux méthodes utilisées actuellement ou que nous croyons applicables dans un avenir rapproché.

Mots-clés : recherche opérationnelle, transport collectif, problèmes d'horaires.

1. INTRODUCTION

The purpose of this paper is to briefly review how operations research techniques have been used to solve planning and operational problems in transit authorities. This review is not exhaustive, but emphasizes operations research techniques that have been applied or that, in our opinion, have potential applications. We are clearly unavoidably biased towards the large amount of work done in this area at the Centre de recherche sur les transports of the Université de Montréal: we apologize if we have overlooked some important work done elsewhere. A further limitation of this paper is that it examines transit problems in isolation from two larger problems in which it is embedded. First, transit is very interrelated with the general urban transportation problem. Street congestion, availability of reserved lanes (part of the road reserved uniquely for public transport), parking policy and pricing, and limits on private car utilization all have a significant impact on transit planning and operations. Secondly, urban development plans may directly influence decisions concerning the location of major transit infrastructures (e.g. subways, lightrail) so as to encourage urban development in the directions desired. This important interrelation is not considered here.

2. THE ELEMENTS OF TRANSIT PROBLEMS

Figure 1 presents the various elements necessary for the planning and operation of a transit service from an operations research perspective. As can be seen, there are many interrelations between the elements of the problem. However, it is not mathematically feasible to take into account all these elements at the same time. It is part of the art of operations research to split large inter-related problems into pieces that are at the same time significant and tractable. These pieces often correspond to the internal organization of transit authorities. Most of the time, strategic

planning activities (introducing new routes, building a subway) are separated from the timetabling and vehicle and crew scheduling parts, which are in turn separated from the day-to-day dispatching of activities. We will describe these problems in more detail in the following sections together with the operations research techniques used in these contexts.

We classify problems according to the various levels of planning. First, strategic planning is related to long term commitments such as subway and light rail development and also modifications to the bus route structure. Because it is difficult for transit users to easily adapt to new bus route structures, any bus route change must be studied carefully before it is introduced and must stay in place for some time to achieve fully the projected impact. As such there is a certain rigidity about the bus route structure, making it part of the strategic planning problem. Operational planning problems are concerned with all the scheduling-related activity, including the production of a timetable for trips, the scheduling and assignment of vehicles, the generation of daily drivers duties according to the various union rules, and the construction of duty rosters composed of the daily duties of individual drivers together with their days off and holidays.

The tactical planning aspect of the problem which is normally related to strategies for operating a given infrastructure and a given fleet are not normally recognized in transit operations. These strategies are either implicitly part of the planning department and become mixed with strategic planning activities, or are included in the operational branch of the transit authority. We will however discuss these problems independently, because we believe that further research is necessary in this area. Finally, we will briefly describe transit dispatching activities.

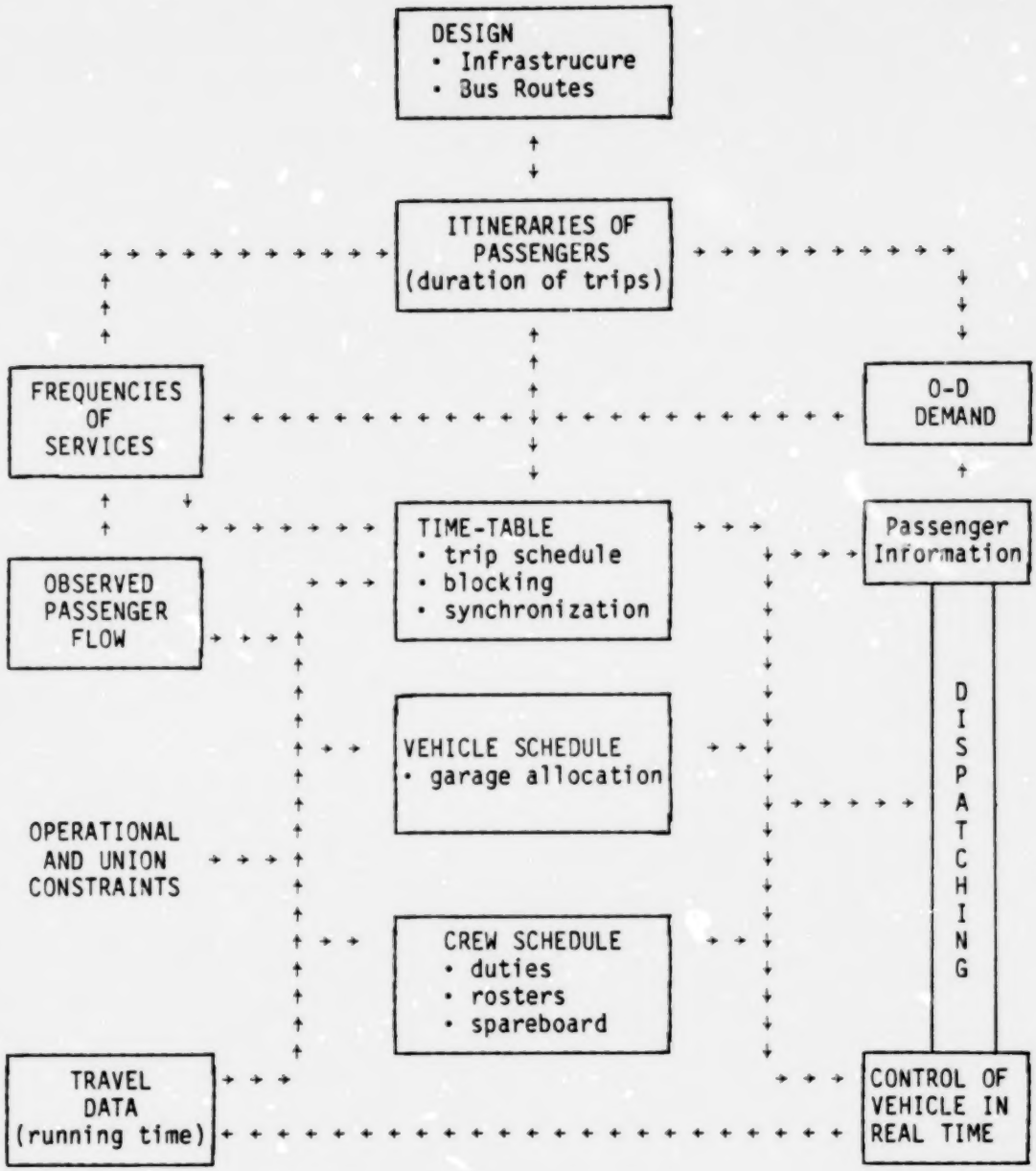


Figure 1 : Main interrelations between the various elements of the transit planning and operations problem.

3. THE STRATEGIC PLANNING PROBLEM

The automatic route design problem for transit is certainly among the most difficult network design problems because of the non-trivial interrelations between the design of routes done by the supplier and the choice of travel itinerary carried out independently by users. We are in a user optimum context rather than a system optimum context because the network planning agency does not decide the actual routing (or routing strategies) of the traffic on its network (which is the case for telephone companies, pipelines, trucking, etc.). In addition, the objective function is ill-defined as in many design problems. The planner certainly wants to minimize the total travel time of the population, but he or she also wants to give "equitable" access to the system to every tax payer, design "attractive routes" that will be "easy" to use, maybe "favour" the use of heavy infrastructure like subways to minimize operating cost, while taking into account so-called "acquired rights" to certain services and taking into consideration political pressures for better services in their electoral district.

For all these reasons, it seemed appropriate to favour an approach by which the design of new infrastructures or new bus routes is done interactively, with the planner choosing scenarios (route structure) while the computer performs the evaluation of the scenarios, providing the planner with information to help in the design of new scenarios, etc.

In the context of automatic design we know of only one approach which is used in practice by a consulting firm VTS (Volvo Transportation System). In this package called VIPS, the approach described in Hasselström (1981) is used. This is essentially a typical location-allocation approach. It is first assumed that there is a direct transportation link between each pair of O-D zones and the frequency of service is calculated accordingly.

The algorithm allocates O-D passengers to minimize travel time. It sequentially drops from the network the least attractive routes, reallocating then the passengers on the remaining routes and recalculating the frequency of services until a satisfactory solution is reached. It is not known how well this system works because it is used mostly by the VTS consultants and has been rarely installed as a planning tool in a transit organization.

On the other hand, there are several computer packages that favour an interactive approach to the design of bus routes and transit infrastructures. Among these, we should mention IGTDS (General Motors, 1980), Transcom (Chapleau, 1974), Transept (Last et al., 1976), NOPTS (Rapp et al., 1976), VIPS (Andreasson, 1976), U load (UMTA, 1977), TRANSPLAN (Osleeb et al., 1976), Madituc (Chapleau et al. 1982) and EMME/2 (Babin et al., 1982). EMME/2 is certainly the most sophisticated evaluation model on the market. It provides extensive graphical displays that assist the user in the evaluation of a transit network. The planner defines interactively its scenarios and the program calculates how the passengers use the system, and provide answers through a series of graphical (and numerical if desired) outputs. It is to be noted that this system provides a total planning environment for urban transportation by letting the user define up to 10 modes of transportation. This system is now used in more than 60 cities around the world and is described in several papers (Babin et al., 1982, 1984; Florian, 1985; Florian and Spiess, 1986).

As an evaluation tool, it supposes that the transit network is given, including the frequency of services on each route. It also supposes that total travel demand by transit between O-D zones is known (e.g. the case where a modal split function has been calibrated to extract the O-D demand for transit). From this information it calculates the itinerary of passengers and the transit flow on each line together with average trip duration between each O-D zone and other local and global measures. In this

context, one feedback loop possible is the reevaluation of matrix by using a mode choice function. The process may be repeated until a stable solution is obtained.

The key operations research problem in any evaluation model, as well as in any heuristic design approach, is to calculate passenger allocations on the network. The key behavioural principle is well accepted; the transit user wants to travel at minimum "inconvenience", this being defined as a mix of waiting time, travel time, travel cost and transfer inconvenience. Several generalized cost functions can be calibrated and tested in each context assigning different weights to the various factors composing it.

However, even given this objective function, the solution of the passenger allocation problem is not as simple as the solution of a shortest path problem (as it might appear at first sight). First, the O-D matrix is an agregation at a zonal level; it is not evident that for two persons living in the same zone and having the same destination, that the same route is the best. Their exact location in the zone may dictate different choices. Second, on a given street segment or at a given intersection, there may be several transit routes passing. The optimal choice of itinerary for a transit user may very well be dictated by the arrival of the first bus on any of these routes. In the same way, the transit user may well change his or her itinerary according to the presence of a given bus at a certain street intersection, etc.

The transit allocation or assignment problem has been studied by several authors, either as a separate problem (e.g. see Dial, 1967,1971; Chriqui, 1974; Chriqui and Robillard, 1975; Rapp et al., 1976; Andresson, 1976; Spiess, 1983) or as a subproblem of more complex models, such as transit network design (see e.g. Lampkin and Saalmans, 1967; Scheele, 1977; Mandl, 1979; and more recently Hasselström, 1981), or multimodal network equilibrium (Florian, 1977; Florian and Spiess, 1983).

All the algorithms proposed, except the one suggested by Spiess, may be classified as heuristic approaches to this problem, and are not based on a behavioral principle. Rather, the algorithms are stated directly and in most cases they involve assignment techniques normally used for car traffic on road networks (e.g. shortest path, multipath assignment), adapted to reflect some of the phenomena inherent to transit networks. They seem however to produce acceptable solutions and are widely used.

Spiess, however, in his paper has developed a new approach to the fixed route transit assignment problem. There are two main differences between the Spiess model and other approaches. First and most important, in his model the traveller does not choose a path, as in all existing methods, but more realistically he selects a strategy. While in all existing methods the transit traveller's route choice is limited to one path, defined as a sequence of route segments, links or transfer points, the strategies considered in Spiess approach allow the transit rider to select any subset of route segments that lead to his or her destination and allow the vehicle that arrives first determine which of the alternative routes is actually taken on an individual trip. This concept of optimal strategies allows more realistic modelling of the traveller's behavior. Second, the transit assignment problem has been formulated for the first time as a mathematical programming problem based on sound behavioral assumptions, and may be solved without recourse to heuristic approaches. In its general form, the model allows for arbitrary vehicle inter-arrival time distributions, but is not computationnally tractable. Using the special case of combining frequencies, the problem size reduces drastically and results in a linear programming formulation that has similarities with the shortest path assignment problem. The proposed solution algorithm solves the problem in polynomial time. Since time and memory requirements of the algorithm are comparable to those of the shortest path assignment, the algorithm may be

applied even to very large transit networks. This method is used in EMME/2. A method based on similar ideas is also being developed and implemented in Torino by Nguyen and Pallottino (1985). The system is called MAESTRO.

The estimation of transit demand is a whole field of research in itself. There has been a great deal of research carried out in this area using aggregated or disaggregated approaches. Surveys of these methods can be found in Florian (1984). The construction by survey of an O-D matrix for urban transportation is a lengthy and costly process. Several research projects are underway to try to estimate these O-D matrices from the observation of flows on the transit network. None of these techniques are yet in operational use, but a very extensive test of this concept is being carried in Torino. A presentation of this approach can be found in Nguyen et al. (1986).

In this general area of transit network design, additional development may come from the automatic generation of options or marginal improvements systems that could be proposed to the planner. These suggestions could be generated through a set of rules gathered together in an expert system by the planners themselves. However, we do not believe that it is realistic to envisage a totally automatic design tool for the transit problem in the near future, expert system or not.

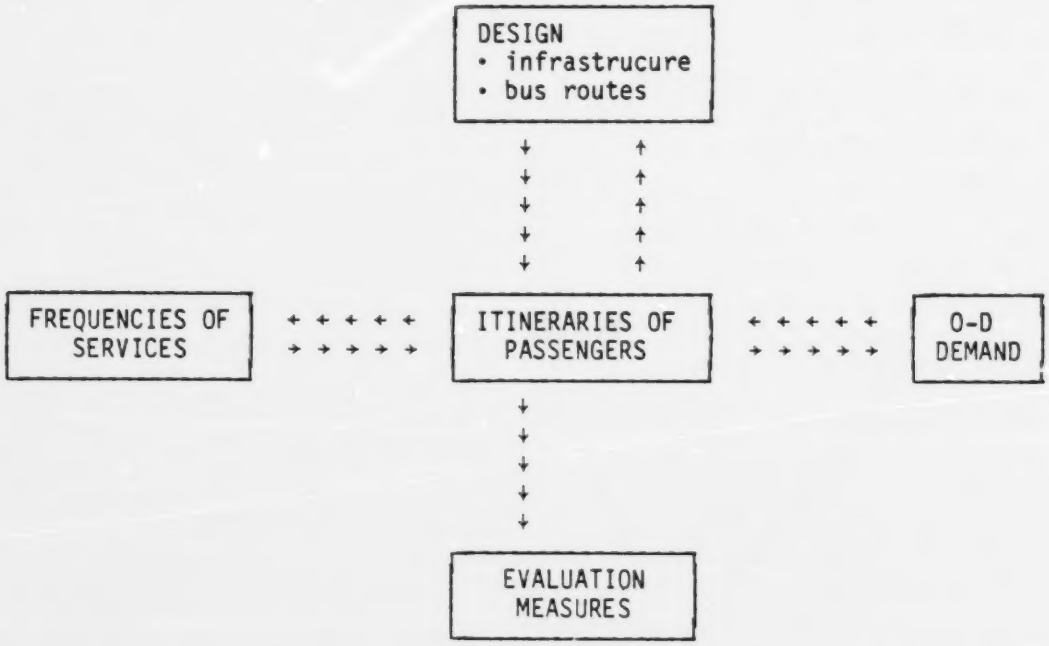


Figure 2 : The strategic planning problem

4. TACTICAL PLANNING

The tactical planning problem in the transit context has barely been studied. Figure 3 summarizes the various elements of this problem. The transit network is assumed known. The objective is to try to find out how to best utilize the available resources (buses and drivers) in order to provide the best service possible at the least cost. Several problems can be studied at this level, but here again the transit assignment problem is at the center of each of these problems. One problem that has received some attention at least partially (Fearnside et al., 1971; Last et al., 1976; Scheêle, 1977) is the problem of choosing the optimum frequency of bus routes, given a fix fleet of vehicles in order to minimize total travel time. The frequency of services influences the itinerary and duration of travel, which in turn may influence travel demand.

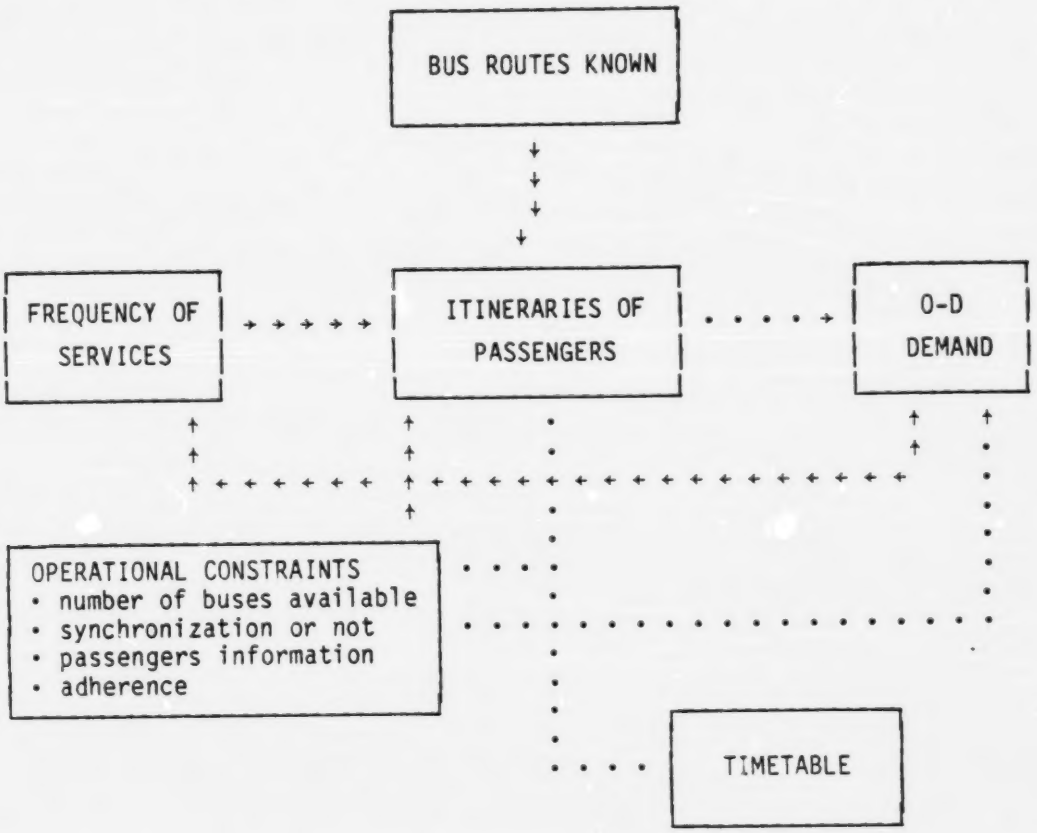


Figure 3: Tactical planning level

There are two fundamentally different ways of defining the frequency of services in a transit network. One is inspired by industrial engineering techniques. A level of service is defined with a given acceptable number of passengers per vehicle at the peak loading point and the frequency of services is increased or decreased accordingly. If more passengers start to use a route, its frequency of service is increased, which may attract more passengers. On the contrary, if the number of passengers on a route decreases, its frequency is decreased until a predetermined minimum level of services is reached. The second approach is a product oriented approach. This approach defines, for example, some routes to be serviced every 10 minutes, others to be serviced every 20 minutes, and maybe others to be

serviced every 30 minutes. The frequency may vary during the day (for example, 10 minutes at peak, 20 minutes off-peak), but the level of service is known, and advertised, and the public may thus know the timetable with some precision. The transit schedule is thus a product that is marketed and advertised, so that passengers may reduce their waiting time. Additional services or articulated vehicles may be added where the number of passengers require it. The respective impacts of respective strategies on travel behavior is not well known.

In addition, we do not know what impact passenger information systems might have in suggesting to transit riders their best travel itinerary depending on the time at which they initiate travel, together with strategies to synchronize bus timetables in order to minimize transfer time. Technologically, it is now feasible to control in real time the adherence of buses to the schedule and to inform the passenger of the time of the next bus through direct access to the company data bank by telephone and voice synthesizer, or with a terminal.

The presence of advanced technology has created a whole new set of problems to be studied. This may eventually change completely the way transit authorities plan their services. In the future we may have to plan bus routes and their schedules under the assumption that the user will know when to show up at a bus stop and which itinerary to choose to minimize his or her travel time.

5. OPERATIONAL PLANNING MODELS

In operational planning, we consider that the requirement for services on each route is known (frequency of service), that the running time tables (time to travel between reference points on a route) are known for each period of the day, and all operating and union constraints are fixed. We then have to define timetables, vehicle schedules and the crew schedules.

5.1 Timetabling

Given a requirement for services and the running timetable, the generation of trips is immediate. Combining trips into blocks is generally fairly trivial. After ensuring that a minimum layover is allowed to maintain the regularity of services (Wilson, 1986), a first-in first-out (FIFO) strategy is optimum on a route by route basis both for the number of vehicles necessary, and for the minimization of total duty times (Saha, 1970). A vehicle is freed as soon as it is no longer required or would have to wait an excessive amount of time for the next trip assignment. A vehicle is dispatched from the depot when needed for the trip schedule.

There are very few mathematical considerations in timetable preparation, but there is a need for a sophisticated management information system. Several bus scheduling software packages are available, most of these integrated into a crew scheduling software (see Wren, 1981; Rousseau, 1985).

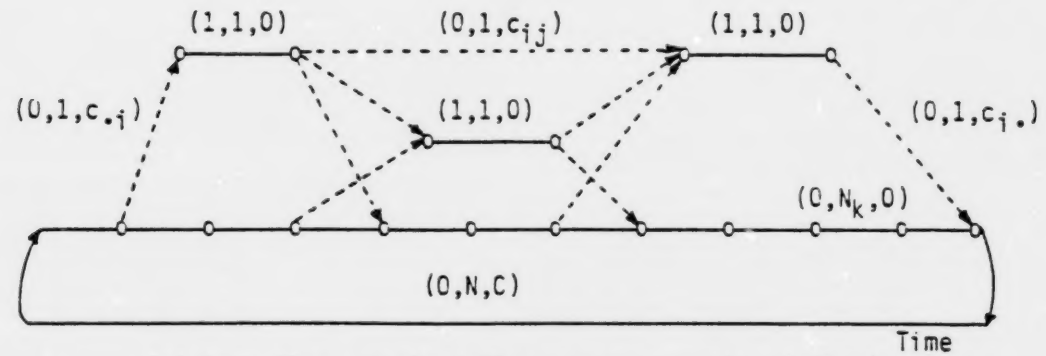
5.2 Vehicle Scheduling Problems

The simple case

By vehicle scheduling, we refer to the class of problems in which vehicles must be assigned to time-tabled trips, or tasks in such a way that each task is carried out by one vehicle, a given set of constraints is satisfied and a cost is minimized. For simplicity here, the term "trip" will be used to denote all vehicle tasks defined in the time-tabling activity (whether trips or blocks of trips). Very often, the cost structure reflects the fact that the number of vehicles must be minimized. We assume that for each trip, the departure time, trip duration, and origin and destination terminal are known. In addition to regular trips, we assume that deadheading trips are allowed between terminals.

The problem where no deadhead trips are allowed was addressed in the previous section when we discussed the formation of blocks. In this section we will discuss the problem as stated above with one depot. We will then consider in turn some complicating constraints. The reader who requires more details on this problem could consult Scott (1986), Carraresi and Gallo (1984b), Bodin et al. (1983) and Ceder and Stern (1981). In his recent paper Scott (1986) presents an excellent survey of the development of approaches for this problem.

Desrosiers et al. (1982) have presented a scheduling model which takes into account the use of a single vehicle to serve many chains of trips. Figure 4 describes the approach proposed. In this approach, each trip in the schedule is represented by an arc with a minimal and maximal flow of one, thus requiring a unit flow to traverse it. There is no specific cost associated with the trips themselves because they all have to be



(minimum flow, maximum flow, cost per unit of flow)

o——o : correspond to trips
- - - - : deadhead trip to or from the depot or between regular trips

Figure 4 : A time space network for the transit bus scheduling problem.

used. All feasible deadhead trips are also represented by arcs with a minimum flow of zero and maximum flow of one and an associated cost c_{ij} corresponding to the cost of deadhead time and distance. Equivalently, a cost is associated with a deadhead trip to and from the garage ($c_{.i}$,$c_{.i}$). The originality of this formula tion lies in the fact that the garage is not represented by a single node but by a sequence of time nodes. All buses in the garage can flow freely from a time node to the next time node without cost. A bound N_k on the maximumnumber of buses in the garage at time period k can be imposed easily on the solution. Finally a return arc from the end of the day to the beginning of the day ensures the conservation of flow and counts the number of buses required to operate the schedule. The fixed cost C of a vehicle and an upper bound N on the number of vehicles can be imposed on this arc.

A standard minimum cost flow algorithm can be used to solve this problem. This formulation accurately takes into account the number of buses required without generating too many arcs.

However Scott (1986) has demonstrated recently that if deadhead cost is proportional to travel time and waiting time, which is characteristic of a broad class of vehicle scheduling problems, there need not be any conflict between the two objectives of minimizing fleet size and minimizing the cost of operating a schedule. In other words, there is never, in general, any need for the application of a fleet size penalty to the creation of independent chains when one intends to find a minimum-cost scheduling solution which uses a minimal fleet.

Complicating constraints

We will consider here three types of complicating constraints that can destroy the structure of the basic vehicle schedule observed earlier. First, we may want to impose a minimum length or a maximum length on the

duration of a vehicle duty (set of trips assigned to a vehicle between its visits to the garage). The minimum length constraint arises in the context where there is a minimum duration for each piece of work assigned to a driver, for example 2 hours. In this context, a vehicle duty of duration less than two hours will cost a minimum of two hours of driving time. It is thus advisable to try to reduce the number of such vehicle duties as far as possible. The maximum duration constraint is in general related to the necessity of refuelling vehicles.

Both of these problems have to be dealt with heuristically. In practice the following heuristics have been found useful :

Step 0. Solve the problem without these maximum or minimum length constraints.

Step 1. Split the day into a certain number of time periods.

Step 2. For each time period split the vehicle duty into two partial duties: set of trips finishing before or during the time period, and the set of trips finishing after the time period. Solve an assignment problem between the two sets of partial duties taking into account the cost of having vehicle duty durations inferior to the lower limit, and prohibiting duties from being longer than the upper limit.

Step 3. Cycle through the time period or various sets of time periods until no improvement is possible.

The second set of complicating constraints arises because one may want to take advantage of a certain level of flexibility in the time-table concerning the departure time of the trip (for example, trip departure time may vary within ± 5 minutes of a desired time). Desrosiers et al. (1984) have described a combinatorial approach for this type of problem in which they solve a set covering problem with column generation over all possible feasible vehicle duties respecting the time windows. These vehicle duties

are not predefined but are generated as the algorithm progresses using shortest path or dynamic programming type algorithms.

A third set of constraints occurs for companies that operate their fleet out of several garages each with a preset capacity. In this case, a vehicle belonging to one garage must return to the same garage. Additional constraints may also be imposed. The company may want to use each garage at a certain minimum capacity, even if this is not cost effective as is very often the case with new garages that have to be located in industrial rather than residential areas. The company may also require that a certain number of vehicles be always out of the garage to facilitate maintenance activities. Finally for control reasons, the company may not accept that the vehicles assigned to a route be split between several garages.

Various approaches have been proposed for this problem (Bodin et al., 1983; Mandle, 1980; Carraresi and Gallo, 1984a; Désilet and Lessard, 1984; Maze et al., 1985). Essentially, a choice must be made between clustering the trips and assigning them to the garage first, and then scheduling vehicles in each garage separately, or first scheduling the whole fleet as if there were only one garage and then assigning the resulting trips to each garage. If there are no constraints on separating a given route among several garages, Carraresi and Gallo (1984a) have proposed an approach based on a Lagrangean relaxation of the problem coupled with a subgradient optimization technique to find an optimal solution. On the other hand, Désilet and Lessard (1984) have proposed a simple clustering heuristic that can take into account all kinds of operational constraints. They sequentially assign all the trips of each route to the best garage and then search for a feasible solution by neighbourhood exchange heuristics, moving routes from one garage to another in order to improve the cost or feasibility in relation to the various constraints. They then optimize the schedule for each garage. These methods can also be used to study the location of new garages and evaluate their impact on operational costs.

Finally, although crew costs often represent more than 60% of operating costs, it has not been found very useful in general to take into account the crew scheduling constraints in preparing the vehicle schedule. The minimum length of a piece of work seems to be the only important constraint to be taken into account at this step. We believe that only in very specific contexts such as small problems with particular constraints would it be profitable to take these constraints into account. This problem has been studied by Scott (1984).

5.3 Crew Scheduling

Crew schedule planning in a transit company involves two different operations: the generation of the drivers' daily duties and the assignment of a set of daily duties (called a roster) to each driver. We will consider each of these problems in turn. The proceedings of the last two workshops on computer scheduling of public transport constitute an excellent set of references on the subject (Wren, 1981; Rousseau, 1985).

Duty Generation

The problem can be expressed simply as the creation of a set of duties of minimum cost ensuring that each bus trip is covered by a driver (or a crew), and that all union contract rules are respected. Several authors favour an approach based on the set covering formulation of the problem. Theoretically, in this approach all feasible duties have to be generated according to union contract rules and the least cost subset of duties that covers all bus trips is mathematically selected.

If the operating or union rules restrict a driver to working on only one bus route, and if there are not too many feasible duties due to strict working rules, this approach may be feasible. Otherwise, if for example

full interlining[1] is permitted, the number of feasible duties is extremely large and simply cannot be generated. Advocates of the set covering approach then have to either split the problem into several subproblems (evening problem, morning problem, etc., see Ward et al.,1981) or heavily restrict the generation of duties according to heuristic rules. This is the case of Busman, Ramcutter, Crewsched, etc. (see Rousseau, 1985, for several papers based on this approach).

The difficulty with this approach is mainly that there does not exist a good set of rules for preselecting the good feasible duties. A typical duty may be very efficient under certain operating and union rules while being very inefficient under another set of rules. A very interesting study has been completed recently by Desrochers (1986) where, while using the set covering approach for the problem, he sequentially generates only the duties useful to improve the solution at each step of the algorithm. Thus only a small subset of the duties have to be generated. It seems however that the technique will apply to only small to medium size problems but could be quite useful in the context where the drivers have to work on only one bus route.

The most popular approach for transit crew scheduling proceeds as follows.

Firstly, the vehicle duties, called blocks[2] in the literature, are split into pieces of work[3]. Secondly, the pieces of work are matched together to form driver duties, and thirdly the solution is improved

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1 Interlining for drivers means that they might work on several bus routes during a single day's operation. Interlining for buses means that buses may change routes within the same vehicle duty.

2 In the crew scheduling problem, the block is generally defined as the itinerary of a vehicle between its departure from and its return to the garage. It includes all deadhead times to take the bus out of and into the garage to and from the route(s) served.

3 A piece of work refers to the period of time during which a driver works continuously with the same vehicle without a break. Generally the number of pieces of work in a duty is limited (2-3 pieces).

heuristically and manually. A matching or an assignment problem formulation for the construction of duties is generally used. Various strategies are possible for splitting the blocks into pieces of work. Most approaches use heuristics adapted to the type of problem considered. In RUCUS (Wilhelm, 1975), the first general package developed for transit crew scheduling, a set of parameters enables the user to partially determine the method by which the pieces will be defined. In Sage runcutting and RUCUS II (Luedtke, 1985), an interactive system for transit crew scheduling, the user has direct control on the ways the pieces are identified. Hoffstadt (1981) and Parker and Smith (1981) include heuristic approaches adapted to their respective problems related to German and British working conditions. The HASTUS system uses a relaxation of the problem to determine how to define the pieces. Ball et al. (1981) form the pieces and blocks by sequentially matching the trips.

While using this general decomposition approach, HASTUS uses a mathematical model to split the blocks into pieces of work. A full description of the algorithm can be found in Rousseau et al. (1985). The authors chose to solve a relaxation of the crew scheduling problem first, trying to keep the main features of the problem while relaxing all the details.

A relaxation of the drivers' scheduling problem is solved on a modified bus schedule. The idea is to retain the essentials of the bus schedule structure and its impact on the contract rules and costs without retaining the precise bus schedule.

Firstly, periods defined by the user are introduced. These periods are normally 30 minutes long; they can vary, depending on the problem, between 15 and 60 minutes. The schedules of all buses are approximated in order that all buses start their block at the beginning of a period and complete it at the end of a period. The authors further assume that relief

times[1] can only occur at the beginning of a period. Moreover, the problem is further relaxed by requiring that the duties selected be sufficient to cover the total requirement of drivers per period instead of requiring that they exactly cover all the blocks individually.

All feasible duties made up of two or three pieces of work covering an integer number of periods and satisfying the union contract are generated. The generating rules eliminate, under instructions from the user, duties which are of no interest or which do not correspond to unwritten rules or common practice (for example, a duty with a long break during a peak hour). All the duties are costed according to the union contract in use. A linear program then searches for the set of duties that will provide the number of drivers required during each period and satisfy other additional constraints at minimum cost.

One type of additional constraint is related to the union contract. All the constraints related to the total number of duties of a certain type are considered. These include constraints on the minimum or maximum number of duties without break and constraints on the maximum number of duties with certain undesirable characteristics like long spread[2], small breaks, etc.

The relaxation is called HASTUS-macro and can also be used as a planning tool for the evaluation of the economic impact of changes in the union contract or in the service level. By changing the cost parameters or

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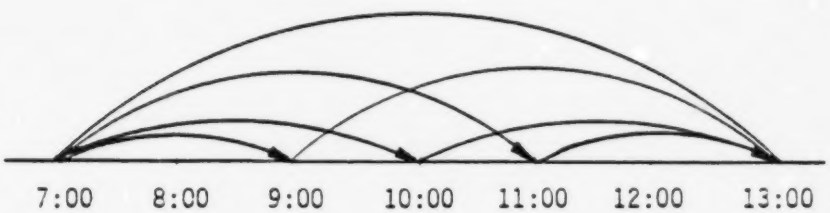
[1] Relief times are the times corresponding to places on a route where a change of drivers is possible. In general the number of such places on each route is small.

[2] The spread is the total length of the duty from the sign on to the sign off. It includes all unpaid breaks.
Example : duty - 1rst piece of work - 6:00 to 10:00
- 2nd piece of work - 14:00 to 18:00
The spread is 12 hours.

the service level a new solution is obtained and can be compared with the current solution. The paper by Blais and Rousseau (1982) describes in detail the HASTUS-macro system and its use in that context. In practice, this relaxation has been found to be very efficient, producing cost estimates within 2% of the real operating costs for drivers.

The solution of HASTUS-macro gives an indication of the types of duties that should be constructed in a final solution. The next step uses this information to split the blocks into pieces of work that correspond as closely as possible to the ones generated in the relaxation HASTUS-macro.

Each block is sequentially split into pieces of work by solving a shortest path problem. It is interesting to note that the problem of splitting a block into pieces can be expressed as the problem of finding a path from the starting time of the block to its ending time using arcs that correspond to pieces of work. Figure 5 illustrates this concept.



The feasible pieces are

7 - 9	7 - 11	9 - 11	10 - 13
7 - 10	7 - 13	9 - 13	11 - 13

Figure 5 : The flow formulation for the partition of a block.

The feasible pieces of work (we are assuming here a minimum length of two hours) are represented by arcs and listed in the figure. We have assumed for simplicity that a relief time exists at every hour in this example.

The cost associated with each piece of work in the flow formulation corresponds to a penalty indicating whether this given piece will increase or decrease the difference between the number of pieces of work of that type desired, and the number of pieces of work of that type already present in the current solution. We cycle through all blocks until no improvement can be achieved towards coming closer to the number of desired pieces of each type indicated by HASTUS-macro.

Once the blocks have been split into pieces of work, a matching algorithm is used to generate duties (Rousseau et al., 1985; Lessard et al., 1985) and several heuristics are used to improve on the solution. The whole system is embedded in a user friendly software (Rousseau and Blais, 1985) and is installed in more than twenty cities around the world (Montreal, Quebec, Calgary, Seattle, New York, Boston, Stockholm, Singapore, Newcastle, Barcelona, ...). The typical savings of this system over a manual solution is of the order of 15% to 20% of unproductive costs (unproductive time, overtime, special payments, etc.), which could represent 2-3% of total operating costs (for example, approximately 2 million dollars of savings per year in Montreal).

The rostering problem

A roster can be defined as the set of duties assigned to a particular driver. Making up rosters may be easy or difficult depending on the particular union contract. In most North American companies the assignment of duties is left to the drivers themselves and the decision is made on a seniority basis : this leaves little or no room at all for mathematical modelling! On the other hand, with many European companies, work load must

be distributed evenly among the drivers. In fact in these problems, few penalties are associated with unpleasant duty conditions. Rather, these duties are evenly distributed among all drivers, who all receive approximately the same monthly pay. The rules for making monthly, weekly or in some cases several month rosters vary widely from one company to another. While there seems to be no general package available for rostering problems in transit, there is a consensus on the most efficient methods to be used to build such rosters. A more detailed description of the approach used can be found in Belleti et al. (1985), Hagberg (1985) and Rousseau (1984).

The basic method is a heuristic based on a construction phase and on a marginal improvement phase. The construction phase can be described as follows : given a set of partial rosters P_i constructed from day 1 to day i (initially the set of partial rosters is composed of all duties for day 1), construct P_{i+1} by defining an assignment problem between the partial rosters in P_i and the duties that have to be performed on day i+1.

The assignment problem can be solved with standard well known algorithms. The objective function is a penalty function measuring how the resulting partial rosters will differ on various criteria from the ideal partial rosters. For example, if the main criterion is to equalize as much as possible the total number of driving hours among the rosters, the penalty incurred by assigning duty k to partial roster j might be the square of the difference between the number of driving hours in the resulting roster and the desired level for such a number after i+1 days. A general penalty function is described in Nicolletti (1973 and 1975) for the airline rostering problem.

As for the marginal improvement, all kinds of exchange algorithms well-known in routing and scheduling can be used in this context. An interesting approach however has been applied by Carraresi and Gallo (1984)

for the transit crew rostering problem in which the objective is to minimize the maximum total driving time in the period considered. This problem is described as the multi-level bottleneck problem and it is shown that the problem is NP-complete. An asymptotically optimal algorithm is presented for this case. In general the objective function is different but their type of algorithm could easily be adapted to the general situation when starting from a feasible solution.

Sequentially consider each day i in the rostering period.

Option 1 : Remove from each roster the duties of day i. Define and solve an assignment problem between the duties just removed and the partial rosters with the penalty function in use, allowing only feasible assignments.

Option 2 : Define and solve an assignment problem with on one side the partial rosters composed of all the duties starting before day i and on the other side the partial rosters composed of all the duties starting on or after day i. Of course only feasible assignments are allowed and the same cost function as in the construction heuristic is used.

If no improvement in the objective function is obtained after all days in the rostering period have been considered, the process stops. If an improvement has been achieved, another iteration is performed on the period. Based on the results of Carraresi and Gallo on their specific problem, we can expect this algorithm to perform very well on all rostering types of problems even if the theoretical properties demonstrated by the authors may not hold in all situations. These methods obviously need to be adapted to particular situations, but they have been used successfully in several contexts.

6. THE DISPATCHING PROBLEM

In some respect dispatching problems are similar to operational planning problems. Vehicle schedules including additional or chartered trips must be planned, and spare drivers must be allocated to perform pieces of work left by regular drivers on vacation or reporting sick. The same type of technique as that described earlier can be used in this context. The number of spare drivers necessary to guarantee a certain level of adherence to schedule is an interesting problem that has been studied by Koutsopoulos and Wilson (1987) recently. Here again the technology is opening a whole new field related to the optimum control of a vehicle schedule. With an automatic vehicle location and control system (AVLC), the dispatcher now has several strategies for dealing with a vehicle that is delayed : take measures to help the vehicle catch up with the schedule, or delay the following vehicles in order to insure an even frequency and prevent bunching. Priority for buses at intersection may be used to reduce delay and better control adherence to schedules. In fact, with all the information collected with an AVLC system, even the running timetables may be calculated in order to facilitate adherence to schedules. As we can see, there are new and challenging problems ahead in this field.

7. CONCLUSION

This review has been brief, with very little detail. We have demonstrated however that in the transit area, operations research techniques are alive and well. They are being used to assist in the planning and operation of many transit networks around the world. All the problems have not been solved, and even for the ones that have been solved, better techniques could be found. New problems are arising from the implementation of new technology in transit. Better information will be available from the in-vehicle micro-computer and better information will be required by the public in general. There will be plenty of challenging operations research problems in the transit area for many years to come.

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